Summer Review Honors Pre Calculus

Radicals:

To simplify means that 1) no radicand has a perfect square factor and

2) there is no radical in the denominator (rationalize).

Recall the **Product Property** $\sqrt{ab} = \sqrt{a} \bullet \sqrt{b}$ and the **Quotient Property** $\sqrt{\frac{a}{b}} = \frac{\sqrt{a}}{\sqrt{b}}$

Examples: Simplify $\sqrt{24} = \sqrt{4} \bullet \sqrt{6}$ find the perfect square factor

$= 2\sqrt{6}$ simplify

Simplify $\sqrt{\frac{7}{2}} = \frac{\sqrt{7}}{\sqrt{2}} \bullet \frac{\sqrt{2}}{\sqrt{2}}$ multiply numerator & denominator by $\sqrt{2}$

$= \frac{\sqrt{14}}{\sqrt{4}} = \frac{\sqrt{14}}{2}$ multiply straight across and simplify

Complex Numbers:

Form of complex number: $a + bi$

Where a is the real part and the bi is the imaginary part

Always make these substitutions $\sqrt{-1} = i$ and $i^2 = -1$

To simplify: pull out the $\sqrt{-1}$ before performing any operation

Example: $\sqrt{-5} = \sqrt{-1} \bullet \sqrt{5}$ Pull out $\sqrt{-1}$

$= i\sqrt{5}$ Make substitution

Example: $\left(i\sqrt{5}\right)^2 = i\sqrt{5} \bullet i\sqrt{5}$

$= i^2\sqrt{25} = (-1)(5) = -5$

Treat i like any other variable when $+, -, \times, or \div$ (but always simplify $i^2 = -1$)

Example: $2i(3+i) = 2(3i) + 2i(i)$ Distribute

$= 6i + 2i^2$ Simplify

$= 6i + 2(-1)$ Substitute

$= -2 + 6i$ Simplify and rewrite in complex form

Equations of Lines:

Slope-intercept form: $y = mx + b$

Point-slope form: $y - y_1 = m(x - x_1)$

Standard Form: $Ax + By = C$

Vertical line: $x = c$ *(slope is undefined)*

Horizontal line: $y = c$ *(slope is zero)*

Slope: $m = \frac{y_2 - y_1}{x_2 - x_1}$

Systems of Equations:

$$\begin{cases} 3x+y=6 \\ 2x-2y=4 \end{cases}$$

(2, 0)

Substitution:

Solve 1 equation for 1 variable

Rearrange.

Plug into 2nd equation.

Solve for the other variable.

Elimination:

Find opposite coefficients for 1 variable

Multiply equation(s) by constant(s).

Add equations together (lose 1 variable)

Solve for variable.

Then plug answer back into an original equation to solve for the 2nd variable.

$y=6-3x$	Solve 1st equation for y	$6x+2y=12$	Multiply 1st equation by 2
$2x-2(6-3x)=4$	Plug into 2nd equation	$\underline{2x-2y=4}$	coefficients of y are opposite
$2x-12+6x=4$	Distribute	$8x=16$	Add
$8x=16$ and $x=2$	Simplify	$x=2$	Simplify.

Plug x=2 back into the original equation $\begin{array}{l} 6+y=6 \\ y=0 \end{array}$

Exponents:

Recall the following rules of exponents:

1. $a^1=a$ Any number raised to the power of one equals itself.
2. $1^a=1$ One raised to any power is one.
3. $a^0=1$ Any nonzero number raised to the power of zero is one.
4. $a^m \cdot a^n=a^{m+n}$ When multiplying two powers that have the same base, add the exponents.
5. $\frac{a^m}{a^n}=a^{m-n}$ When dividing two powers with the same base, subtract the exponents.
6. $(a^m)^n=a^{mn}$ When a power is raised to another power, multiply the exponents.
7. $a^{-n}=\frac{1}{a^n}$ and $\frac{1}{a^{-n}}=a^n$ Any nonzero number raised to a negative power equals its reciprocal raised to the opposite positive power.

Polynomials:

To add/subtract polynomials, combine like terms.

EX: $8x-3y+6-(6y+4x-9)$ *Distribute the negative through the parantheses.*

$=8x-3y+6-6y-4x+9$ *Combine like terms with similar variables.*

$=8x-4x-3y-6y+6+9$

$=4x-9y+15$

To multiply two binomials, use FOIL.

EX: $(3x-2)(x+4)$ *Multiply the first, outer, inner, and last terms.*

$=3x^2+12x-2x-8$ *Combine like terms together.*

$=3x^2+10x-8$

Factoring:

Follow these steps in order to factor polynomials.

STEP 1: Look for a GCF in ALL of the terms.

a) If you have one (other than 1) factor it out.

b) If you don't have one move on to STEP 2

STEP 2: How many terms does the polynomial have?

2 Terms a) is it the difference of two squares? $a^2-b^2=(a+b)(a-b)$

EX: $x^2-25=(x+5)(x-5)$

b) Is it the sum or difference of two cubes?

$$a^3-b^3=(a-b)(a^2+ab+b^2)$$
$$a^3+b^3=(a+b)(a^2-ab+b^2)$$

EX:
$$m^3+64=(m+4)(m^2-4m+16)$$
$$p^3-125=(p-5)(p^2+5p+25)$$

3 Terms **EX:**

$x^2+bx+c=(x+\_)(x+\_)$ $x^2+7x+12=(x+3)(x+4)$

$x^2-bx-c=(x-\_)(x-\_)$ $x^2-5x+4=(x-1)(x-4)$

$x^2+bx-c=(x-\_)(x+\_)$ $x^2+6x-16=(x-2)(x+8)$

$x^2-bx-c=(x-\_)(x+\_)$ $x^2-2x-24=(x-6)(x+4)$

Factoring continued

4 Terms---Factor by Grouping

a) Pair up first two terms and last two terms.

b) Factor out GCF of each pair of numbers.

c) Factor out front parentheses that the terms have in common.

d) Put leftover terms in parentheses.

$$Ex: x^3+3x^2+9x+27=\left(x^3+3x^2\right)+\left(9x+27\right)$$

$$=x^2(x+3)+9(x+3)$$

$$=(x+3)\left(x^2+9\right)$$

Solving Quadratics

To solve quadratic equations, try to factor first and set each factor equal to zero. Solve for your variable. If the quadratic does NOT factor, use the quadratic formula.

EX: $x^2-4x=21$ *Set equal to zero FIRST.*

$x^2-4x-21=0$ *Now factor.*

$(x+3)(x-7)=0$ *Set each factor equal to zero.*

$x+3=0 \quad x-7=0$ *Solve for each x.*

$x=-3 \quad x=7$

QUADRATIC FORMULA—allows you to solve any quadratic for all its real and imaginary roots.

$5x^2-2x+4=0$ $x=\dfrac{-b\pm\sqrt{b^2-4ac}}{2a}$

EX: In the equation $x^2+2x+3=0$, find the value of the discriminant, describe the nature of the roots, then solve.

$x^2+2x+3=0$ *Determine the values of a, b, and c.*

$a=1\ b=2\ c=3$ *Find the discriminant.*

$D=2^2-4\cdot1\cdot3$

$D=4-12$

$D=-8$ *There are two imaginary roots.*

$$Solve: \ x=\frac{-2\pm\sqrt{-8}}{2}$$

$$x=\frac{-2\pm 2i\sqrt{2}}{2}$$

$$x=-1\pm i\sqrt{2}$$

Composition and Inverses of Functions:

Recall: $(f \circ g)(x) = f(g(x))$ OR $f[g(x)]$ read **"f of g of x"** means to plug the inside function in for x in the outside function.

Example: Given $f(x) = 2x^2 + 1$ *and* $g(x) = x - 4$ find $f(g(x))$.

$$\begin{aligned} f(g(x)) &= f(x-4) \\ &= 2(x-4)^2 + 1 \\ &= 2(x^2 - 8x + 16) + 1 \\ &= 2x^2 - 16x + 32 + 1 \\ f(g(x)) &= 2x^2 - 16x + 33 \end{aligned}$$

To find an inverse, switch x and y and solve for y.

Example:	$f(x) = \sqrt[3]{x+1}$	Rewrite *f(x)* as *y*
	$y = \sqrt[3]{x+1}$	Switch *x* and *y*
	$x = \sqrt[3]{y+1}$	Solve for your new *y*
	$(x)^3 = \left(\sqrt[3]{y+1}\right)^3$	Cube both sides
	$x^3 = y + 1$	Simplify
	$y = x^3 - 1$	Solve for *y*
	$f^{-1}(x) = x^3 - 1$	Rewrite in inverse notation

Find the inverse, $f^{-1}(x)$, if possible.

* Rationals *

Multiplying and Dividing: **Factor numerator and denominator completely. Cancel any common factors in the top and bottom. If dividing, change divide to multiply and flip the second fraction.**

EX: $\dfrac{x^2+10x+21}{5-4x-x^2} \bullet \dfrac{x^2+2x-15}{x^3+4x^2-21x}$ *Factor everything completely.*

$= \dfrac{(x+7)(x+3)}{(5+x)(1-x)} \bullet \dfrac{(x+5)(x-3)}{x(x-3)(x+7)}$ *Cancel out common factors in the top and bottom.*

$= \dfrac{(x+3)}{x(1-x)}$ *Simplify.*

Rationals continued

Addition and Subtraction

First find the least common denominator. Write each fraction with that LCD. Add/subtract numerators as indicated and leave the denominators as they are.

EX: $\dfrac{3x+1}{x^2+2x}+\dfrac{5x-4}{2x+4}$ *Factor denominator completely.*

$\dfrac{3x+1}{x(x+2)}+\dfrac{5x-4}{2(x+2)}$ *Find LCD, which is* $(2x)(x+2)$

$\dfrac{2(3x+1)}{2x(x+2)}+\dfrac{x(5x-4)}{2x(x+2)}$ *Rewrite each fraction with the LCD in the denominator.*

$\dfrac{6x+2+5x^2-4x}{2x(x+2)}$ *Write as one fraction.*

$\dfrac{5x^2+2x+2}{2x(x+2)}$ *Combine like terms.*

Complex Fractions: Eliminate complex fractions by multiplying the numerator and denominator by the LCD of each of the small fractions. Then simplify the result.

EX: $\dfrac{1+\frac{1}{a}}{\frac{2}{a^2}-1}$ *Find LCD:* a^2

$=\dfrac{\left(1+\frac{1}{a}\right)\bullet a^2}{\left(\frac{2}{a^2}-1\right)\bullet a^2}$ *Multiply top and bottom by LCD.*

$=\dfrac{a^2+a}{2-a^2}$ *Factor and simplify if possible.*

$=\dfrac{a(a+1)}{2-a^2}$

Evaluations

To evaluate a function for the given value, simply plug the value into the function for x.

ex) $f(x)=x^2-6x+3$

$f(-3)=(-3)^2-6(-3)+3$

$=9+18+3$

$=30$

Solving Rational Equations:

Multiply each term by the LCD of all the fractions. This should eliminate all of our fractions. Then solve the equation as usual.

$\frac{5}{x+2}+\frac{1}{x}=\frac{5}{x}$ *Find LCD first* $x(x+2)$

$x(x+2)\frac{5}{x+2}+x(x+2)\frac{1}{x}=\frac{5}{x}x(x+2)$ *Multiply each term by the LCD.*

$5x+1(x+2)=5(x+2)$ *Simplify and solve.*

$5x+x+2=5x+10$

$6x+2=5x+10$

$x=8$ ← *Check your answer! Sometimes they do not check!*

Check: $\frac{5}{8+2}+\frac{1}{8}=\frac{5}{8}$

$\frac{5}{10}+\frac{1}{8}=\frac{5}{8}$

$\frac{5}{8}=\frac{5}{8}$

Synthetic and Long division

Synthetic should only be used when dividing by a "linear" polynomial.

Ex $\frac{2x^3+3x^2-6x+10}{x+3}$

Synthetic

$$\begin{array}{r|rrrr} -3 & 2 & 3 & -6 & 10 \\ + & & -6 & 9 & -9 \\ \hline * & 2 & -3 & 3 & \boxed{1} \end{array}$$

$2x^2-3x+3+\frac{1}{x+3}$

Long Division

$$\begin{array}{r l} & 2x^2-3x+3 \\ x+3 \,\big) & 2x^3+3x^2-6x+10 \\ & -(2x^3+6x^2) \\ \hline & -3x^2-6x \\ & -(-3x^2-9x) \\ \hline & 3x+10 \\ & -(3x+9) \\ \hline & 1 \end{array}$$

$\frac{2x^3}{x}=2x^2$

$\frac{-3x^2}{x}=-3x$

$\frac{3x}{x}=3$

$2x^2-3x+3+\frac{1}{x+3}$

All work should be shown and completed on separate paper. Answers are provided.

1 - 4 Perform the indicated operation.

1) $\frac{x}{2} - \frac{2x}{5}$	2) $\frac{3}{2x} + \frac{4}{x^2}$	3) $\frac{2}{7x} \cdot \frac{x^3}{8}$	4) $\frac{2}{7x} \div \frac{x^3}{8}$

5 - 8 Simplify. Answers should only have positive exponents.

5) $x^2 \cdot x^3$	6) $(x^2)^3$	7) $(3x^2y)^{-2}$	8) $\frac{x^{-1}y}{xy^{-2}}$

9 - 13 Simplify each expression. (exact values - no decimals)

9) $5\sqrt{40}$	10) $2\sqrt{3} \cdot 2\sqrt{6}$	11) $\sqrt{-49}$	12) $(3-4i)^2$	13) $(1+\sqrt{3})(1-\sqrt{3})$

14 - 19 Factor each polynomial completely.

14)$x^2 - 4x - 21$	15) $6x^2 - 7x - 3$	16) $x^4 - 81$
17)$y^4 + 6y^2 + 9$	18) $2x^4 - 6x^2 + 8$	19) $3y^2 - 75$

20 - 21 Solve each equation for y.

20) $7y + 6x = 10$	21) $\frac{1}{4}y - 7x = \frac{15}{2}$

22 - 23 Find the solution of the system of equations. Write the answer in the form of (x, y). Use either substitution or elimination.

22) $-2x - 5y = 7$ $7x + y = -8$	23) $4x + 9y = 2$ $2x + 6y = 1$

24 - 28 Find an equation in slope-intercept form of the line described.

24) The line through (3, - 2) with slope of $\frac{4}{5}$
25) The line through the points (-1, - 4) and (3, 2)
26) The line through (- 2, 4) with slope of 0.
27) The line through (2, - 3) and parallel to the line $2x + 5y = 3$
28) The line through (2, -3) and perpendicular to the line $2x + 5y = 3$

29 - 33 **Given $f(x) = x^2 - 4$ and $g(x) = \sqrt{2x + 4}$, determine each of the following.**

29) $f(3)$	30) $f(g(4))$	31) $f(g(x))$
32) Domain of $f(g(x))$	33) $f^{-1}(x)$	

34 - 35 Simplify. Leave answers in "general form". (Highest degree —> lowest degree)

34) $3x^3 + 9 + 7x^2 - x^3$	35) $7m - 6 - (2m + 5)$

36 - 38 Multiply. Leave answers in "general form".

36) $(3a + 1)(a - 2)$	37) $(c - 5)^2$	38) $(5x + 7y)(5x - 7y)$

39 - 43 Factor and Simplify.

39) $\frac{5z^3+z^2-z}{3z}$	40) $\frac{m^2-25}{m^2+5m}$	41) $\frac{10r^5}{21s^2} \cdot \frac{3s}{5r^3}$

42) $\frac{x^2+10x+24}{x^2+11x+28} \div \frac{x^2+6x}{x^2-x-56}$	43) $\frac{2-a^2}{a^2+a} + \frac{3a+4}{3a+3}$

44 - 46 Solve. Check your solutions by plugging into the original equation.

44) $\frac{8}{x+3} - \frac{3}{x-3} = \frac{15}{x^2-9}$	45) $\frac{x+10}{x^2-2} = \frac{4}{x}$	46) $\frac{5}{x-5} = \frac{x}{x-5} - 1$

47) Solve by Completing the Square: $x^2 - 5x - 3 = 0$

48) Simplify the complex fraction: $\dfrac{\frac{1}{2} - \frac{x+5}{4}}{\frac{x^2}{2} - \frac{5}{2}}$

Divide each polynomial using long division OR synthetic division.

49) $\frac{c^3-3c^2+18c-16}{c^2+3c-2}$	50) $\frac{x^4-2x^2-x+2}{x+2}$

** Be able to recognize the following parent functions by: equation and graphically. **

A. Linear: $y = x$	B. Quadratic: $y = x^2$	C. Cubic: $y = x^3$
D. Square Root: $y = \sqrt{x}$	E. Absolute Value: $y = \lvert x \rvert$	F. Rational: $y = \frac{1}{x}$

Answers to PreCalc Honors Summer Review

1) $\frac{5-4x}{10}$ 2) $\frac{3x+8}{2x^2}$ 3) $\frac{x^2}{28}$ 4) $\frac{16}{7x^4}$

5) x^5 6) x^6 7) $\frac{1}{9x^4y^2}$ 8) $\frac{y^3}{x^2}$

9) $10\sqrt{10}$ 10) $12\sqrt{2}$ 11) $7i$ 12) $5-24i$ 13) -2

14) $(x-7)(x+3)$ 15) $(2x-3)(3x+1)$ 16) $(x+3)(x-3)(x^2+9)$

17) $(y^2+3)^2$ 18) $2(x^4-3x+4)$ 19) $3(y+5)(y-5)$

20) $y=-\frac{6}{7}x+\frac{10}{7}$ 21) $y=28x+30$

22) $(-1,-1)$ 23) $(\frac{1}{2},0)$

24) $y=\frac{4}{5}x-\frac{22}{5}$ 25) $y=\frac{3}{2}x-\frac{5}{2}$ 26) $y=4$

27) $y=-\frac{2}{5}x-\frac{11}{5}$ 28) $y=\frac{5}{2}x-8$

29) 5 30) 8 31) $2x$ 32) $\mathbb{R}$ 33) $f^{-1}(x)=\pm\sqrt{x+4}$

34) $2x^3+7x^2+9$ 35) $5m-11$ 36) $3a^2-5a-2$

37) $c^2-10c+25$ 38) $25x^2-49y^2$ 39) $\frac{5z^2+z-1}{3}$ or $\frac{5}{3}z^2+\frac{1}{3}z-\frac{1}{3}$

40) $\frac{m-5}{m}$ or $1-\frac{5}{m}$ 41) $\frac{2r^2}{7s}$

42) $\frac{x-8}{x}$ or $1-\frac{8}{x}$ 43) $\frac{2(2a+3)}{3a(a+1)}$ 44) $x=\frac{48}{5}$ 45) $x=-\frac{2}{3}, 4$

46) $\mathbb{R}$ 47) $x=\frac{5}{2}\pm\frac{\sqrt{37}}{2}$ 48) $-\frac{x+3}{2(x^2-5)}$ 49) $c-6+\frac{38c-28}{c^2+3c-2}$

50) $x^3-2x^2+2x-5+\frac{12}{x+2}$