AP Calculus Students: Welcome to AP Calculus.

In this packet, you will find numerous topics that were covered in your Algebra and Pre-Calculus courses. These are topics that I expect you to know thoroughly before you begin AP Calculus this coming school year. If you do not know these topics, you will likely struggle to keep up with the pace of the class.

Each of the more difficult topics has sample problems with worked-out solutions and extra questions at the end of the packet for you to practice, if necessary. The extra questions also have solutions, though they are not worked out. These topics will not be thoroughly covered during a review time at the start of the school year, but you will be tested over them on the 2nd day that we have AP Calculus class.

If you have questions regarding these topics, you may contact me for help throughout the summer at school. I am at school often and will be checking my e-mail regularly. Sometimes, I will be able to help you through e-mail. Otherwise, you may need to find a time to come in to see me before school begins to get help.

The review packet is not a required assignment, but if you come to class next school year without knowing these topics completely, it is likely that you will quickly fall behind in the new material.

I recommend that you look over each topic with its sample questions and then determine which topics you feel you need to review. Then, attempt each section of extra questions for which you feel you need extra practice. If you would like additional practice questions, you may contact me for those as well.

I know that initially this will look like a lot of material, but keep in mind that this is a review of material that you have already learned. If you remember the material, you will be able to move through this packet very quickly.

Please bring this packet with you to class at the beginning of the school year for AP Calculus. I will go over some of the questions if there are multiple students struggling with the same topics. I also recommend that you keep this packet as reference material for use throughout the year.

For the material on limits, you need to watch a YouTube video covering this material and finish the corresponding separate worksheet on limits. This worksheet is a required assignment and is due on the 1st day of class. The video is at www.youtube.com/watch?v=22t_mmcwD9A.

If you have questions, the best way to contact me is by e-mail at school, aoats@hpcacougars.org. You may call the school office and contact me at extension 232 though I am only there at various times. If you would like extra information regarding the AP Calculus course, you may look at the College Board website at www.collegeboard.com and look for information regarding the AP Calculus AB course (not BC).

I am looking forward to working with each of you throughout this coming school year.

Mr. Oats

I. Factoring

A. Factoring a common factor: $6x^3 - 3x^2 + 15x^2y = 3x^2(2x - 1 + 5y)$

B. Factoring with fractional exponents:

$4a^2 + 8a^{3/2} - 12a^5 = 4a^{3/2}(a^{1/2} + 2 - 3a^{7/2})$

C. Factoring with negative exponents (use the lowest power even when it is a negative number):

$3m^2 - 2m^{-4} - m = m^{-4}(3m^6 - 2 - m^5)$

D. Factoring trinomials:

$x^2 - 4x - 12 = (x - 6)(x + 2)$ or $(x + 2)(x - 6)$

$6x^2 + 17x + 12 = (2x + 3)(3x + 4)$

E. Factoring difference of squares:

$x^2 - 49 = (x - 7)(x + 7)$

F. Factoring sum or difference of cubes:

$a^3 + b^3 = (a + b)(a^2 - ab + b^2)$

$27x^6 - 64 = (3x^2)^3 - 4^3 = (3x^2 - 4)(9x^4 + 12x^2 + 16)$

G. Factoring by grouping – typically used with 4 terms, but it will not always work for a polynomial with 4 terms:

$6x^3 - 3x^2 - 4x + 2 = 3x^2(2x - 1) - 2(2x - 1) = (2x - 1)(3x^2 - 2)$

II. Finding roots/zeroes of equations without a calculator

Roots and zeroes are synonymous and are closely related to factors. Factors help to find roots and zeroes and differ mainly by their sign (positive or negative). Roots/zeroes occur when the graph of a function crosses the x-axis which is the same as finding the x-value that results in an answer of zero when the problem is written in function form.

A. Finding simple roots:

$3x - 12 = 0$ $3x = 12$ Answer: $x = 4$

B. Finding roots using difference of squares:

$4x^2 = 64$ $4(x^2 - 16) = 0$ $4(x - 4)(x + 4) = 0$

Answer: $x = 4$ or $x = -4$

Note: Some people will solve by dividing both sides by 4 and then taking a square root. That is acceptable as long as you remember to take both the positive and negative square root. Many people forget the negative answer when using this method.

C. Finding roots of trinomials:

1st Example: $x^2 - 5x = 14$ $x^2 - 5x - 14 = 0$

$(x - 7)(x + 2) = 0$ Answer: $x = 7$ or $x = -2$

2nd Example: $4x^2 = 7x - 3$ $4x^2 - 7x + 3 = 0$

$(4x - 3)(x - 1) = 0$ Answer: $x = 3/4$ or $x = 1$

D. Finding roots using the quadratic formula:

Use the quadratic formula when a trinomial can’t be easily factored.

Reminder: The quadratic formula is $\dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$3x^2 - 4x - 9 = 0$

$$\frac{-(-4) \pm \sqrt{(-4)^2 - 4(3)(-9)}}{2(3)} = \frac{4 \pm \sqrt{16+108}}{6} = \frac{4 \pm \sqrt{124}}{6} = \frac{4 \pm 2\sqrt{31}}{6} = \frac{2 \pm \sqrt{31}}{3}$$

Answer: $x = \dfrac{2 \pm \sqrt{31}}{3}$

E. Finding roots using sum or difference of cubes:

$4x^3 - 108 = 0$ $\quad 4(x^3 - 27) = 0$ $\quad 4(x - 3)(x^2 + 3x + 9) = 0$

You will get x = 3 from the first parentheses. The second parentheses cannot be factored. If you apply the quadratic formula, you will get imaginary answers (the square root of a negative number), and we ignore imaginary answers in AP Calculus AB.

Answer: $x = 3$

Some people will solve by moving the 108 to the right side, dividing by 4, and then taking a cube root. That is acceptable and you cannot take both a positive and a negative cube root. Some people think that they must always get both a positive and a negative answer when taking a root, but that only applies to even roots.

III. Finding roots/zeroes with a calculator

Major concepts include moving all terms to one side of the equals sign and then using your *Calculate* menu on your calculator.
All answers in AP Calculus must be accurate to 3 decimal places.

1st Example: $x^2 - 5x + 3 = 0$

Put function into $Y_1 =$, use *Calculate* menu #2, set left and right boundaries around first possible answer (there is no need to make an accurate guess if your boundaries are good), repeat for 2nd answer

Answer: $x = 0.697$ or $x = 4.303$

2nd Example: $e^x = -x^3 + 5x^2 + 0.9$

Move terms so that all terms are on one side first.

Answer: $x = -0.075$ or $x = 0.319$ or $x = 2.926$

IV. Finding intersection point(s) of two graphs without a calculator

A. Solving a system of linear equations

Find the point of intersection: $3x + y = -2$ and $2x - 4y = -20$

Recognize that these are both lines so there are three options: usually there is only one possible answer, there are an infinite number of answers if these are actually the same line in different forms (such as $2x - 3y = 5$ and $-4x + 6y = -10$; multiply the first equation by -2 to get the second equation), or there are no answers if the lines are parallel. You can solve a system of equations by substitution, elimination, Cramer's Rule (uses matrices), Gaussian elimination (row-echelon), or the inverse matrix method. Substitution and elimination are usually much easier when dealing with only 2 variables, and we usually do not need to solve complex 3 variable systems in AP Calculus AB.
Answer: (-2,4) or $x = -2$, $y = 4$

B. Solving a system of non-linear equations:

Find the point(s) of intersection: $y = x^2 + 3x$ and $y = x + 3$

Set the equations equal to each other ($x^2 + 3x = x + 3$) and then solve like finding roots/zeroes without a calculator. Try to evaluate how many possible answers there will be. In this case, the first equation is a parabola, and the second equation is a line. A line and a parabola can intersect once, twice, or not at all.

Answer: $x = -3$ and $x = 1$

You may have to find corresponding y-values also: When $x = -3$, then $y = 0$, the point $(-3,0)$, and when $x = 1$, then $y = 4$, the point $(1,4)$.

Find the point(s) of intersection: $y = 4x^3 - 5x$ and
$y = -2x^2 - x + 2$

This is a cubic function and a parabola. They can intersect once, twice, or three times. Set functions equal to each other, and solve by factoring using grouping and also factoring difference of squares.

Answer: $(-1,1)$ and $(-0.5,2)$ and $(1,-1)$

V. Finding point(s) of intersection with a calculator

There are 2 methods to do this.

The first (and usually the best when using a calculator) is to put each equation into Y1, Y2, etc. and then use *Calculate* menu #5. Press "ENTER" when cursor is on first desired curve, press "ENTER" again when cursor is on second desired curve (if there are more than 2 curves on the graph, use up/down arrows to move to desired curve), press "ENTER" when cursor is near intersection point. Repeat these steps to find additional points of intersection if there are any (you may need to move the cursor left/right before pressing "ENTER" the third time to find additional points of intersection.

The second method is to move everything to one side of the equals sign and solve for zeroes of the new single equation. This can be confusing for some people because there is only one equation and the zeroes of this equation are the points of intersection of the original equations. Also, this method will not give the correct y-values for each point of intersection. Those would have to be solved separately. This problem makes this method much longer if the y-values are required as part of the answer.

Find the point(s) of intersection: $y = x^4 - 2x^3 + 1$ and $y = x^3 - 3x$

Answer: $(-0.618,1.618)$ and $(-0.414,1.172)$ and $(1.618,-0.618)$ and $(2.414,6.828)$

Find the point(s) of intersection: $y = \log(3x + 1)$ and $y = \sin x$

Make sure your calculator is in radian (not degree) mode.

Answer: $(0.230,0.228)$ and $(2.100,0.863)$

VI. Formulas to know

A. Slope formula $= \dfrac{y_2 - y_1}{x_2 - x_1} = \dfrac{\text{rise}}{\text{run}}$

Find slope between the points $(-6,-5)$ and $(-2,5)$

Answer $= \dfrac{5-(-5)}{-2-(-6)} = \dfrac{10}{4} = \dfrac{5}{2}$

B. Distance formula = $\sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$

Find the distance between the points (4,-3) and (-4,1)

Answer = $\sqrt{(-4-4)^2+(1--3)^2}=\sqrt{(-8)^2+(4)^2}=\sqrt{64+16}=\sqrt{80}=4\sqrt{5}$

VII. Trigonometry

A. Identities

Make sure you know the main trigonometric identity: $\sin^2 x + \cos^2 x = 1$. From there, you should either memorize or be able to quickly develop the other 8 trigonometric identities. The first two are developed by moving terms of that identity: $\sin^2 x = 1 - \cos^2 x$ and $\cos^2 x = 1 - \sin^2 x$.

The next identity is developed by dividing each term of the 1st identity by $\sin^2 x$ giving us $1 + \cot^2 x = \csc^2 x$. We can develop two more by moving terms of this identity: $\cot^2 x = \csc^2 x - 1$ and $1 + \csc^2 x - \cot^2 x$.

The 7th identity is developed by dividing each term of the 1st identity by $\cos^2 x$ giving us $\tan^2 x + 1 = \sec^2 x$. We can develop the last two identities by moving terms of this identity: $\tan^2 x = \sec^2 x - 1$ and $1 = \sec^2 x - \tan^2 x$.

B. Values

Know the values on your unit circle in radians – we do not use degrees in Calculus.

C. Properties

This includes things such as period, amplitude, horizontal shifts, and vertical shifts.

VIII. Line Equations

A. Forms

1. Slope-intercept form: $y=\frac{5}{3}x-4$

   This is the form y = *m*x + *b* where *m* is the slope and *b* is the y-intercept (the point where the line crosses the y-axis).

2. Standard form: 3x – 4y = 9

   This form has x and y on the same side of the equation with the constant on the other side. Also, all numbers must be integers – no fractions or decimals.

3. Point-slope form: $y-2=-\frac{2}{3}(x+4)$ where we know that $m = -\frac{2}{3}$ and we know we have the point (-4,2).

   This form is the easiest and fastest form to use for most problems in AP Calculus AB.

B. Solving linear problems

1. Find the equation of the line that goes through (-2,4) and (3,6).

   Use slope formula to find $m = \frac{2}{5}$. Then, using point-slope form, write the equation (you may choose either point, I chose the first point) $y-4=\frac{2}{5}(x+2)$ which can be simplified to $y-4=\frac{2}{5}x+\frac{4}{5}$

which is finally written as $y=\frac{2}{5}x+\frac{24}{5}$. Sometimes, the answer can be left in the initial point-slope form; sometimes, it is necessary to change to another form.

2. Find the equation of the line that is perpendicular to the line 4x – 3y = 8 and passes through the point (6,-2).

   Change the standard form to slope-intercept form to find that the slope of the original line is $\frac{4}{3}$. Therefore, the slope of the new line is $-\frac{3}{4}$. (Remember that parallel lines have equal slopes and that perpendicular lines have slopes that are opposite reciprocals.) Use point-slope form to find new equation: $y+2=-\frac{3}{4}(x-6)$ which can be changed to $y=-\frac{3}{4}x+\frac{5}{2}$ if necessary.

IX. Functions

A. Common graphs: You should know the basic shapes of common graphs. Later we will deal with transformations to the graphs.

1. Quadratic graph (parabola): $y = x^2 - 4x - 5$

   You should be able to determine the vertex, zeroes (if there are any), direction, and width of a parabola.

   Vertex (x-value) = $-\frac{b}{2a}=-\frac{-4}{2(1)}=2$, so y-value is $2^2 - 4(2) - 5 = -9$

   Zeroes (solve by factoring) = (5,0) and (-1,0)

   Direction (if a is positive, up; if a is negative, down): a = 1,up

   Width (if $|a| > 1$, narrow; if $|a| < 1$, wide; if $|a| = 1$, normal): a = 1, so it is normal (normal width means that if you move 1 unit left or right from the vertex, you move up 1 unit; if you move 2 units right or left from the vertex, you move up 4 units, 3 units left or right means 9 units up, etc.)

2. Cubic function: $y = x^3$

   This graph is centered at the origin (0,0), and moves up to the right and down to the left. The graph has 180° rotational symmetry. If you move 1 unit to the right, you will move up 1 unit. Likewise, 1 unit to the left means down 1 unit. Two units right means up 8 units ($2^3 = 8$); 2 units left, down 8 units, etc.

3. Absolute value: $y = |x|$

   A v-shaped graph with the point at the origin (0,0). The graph moves linearly left and right. The left side has a slope of -1, and the right side has a slope of 1.

4. Square root graph: $y = \sqrt{x}$

   This graph looks like half of a parabola; however, it opens to the right rather than up. The graph is centered at the origin. When you move 1 unit to the right, you move 1 unit up. When you move 4 units to the right, you move 2 units up ($\sqrt{4}=2$), etc. You cannot move left on a square root graph.

5. Cube root graph: $y = \sqrt[3]{x}$

This graph looks like the cubic function; however, it opens to the right and left rather than up and down: it has y = x line symmetry with the cubic function. The graph is centered at the origin. When you move 1 unit to the right, you move 1 unit up; and 1 unit left means 1 unit down. When you move 8 units to the right, you move 2 units up ($\sqrt[3]{8} = 2$); 8 units to the left means 2 units down, etc.

6. Exponential growth/decay: $y = a^x$ (a is a positive number)

The basic form of this graph will always pass through the point (0,1) – (anything raised to the 0 power equals 1). If $a > 1$, the graph will rise sharply as it moves to the right and will flatten out as it moves to the left – the x-axis will be a horizontal asymptote. If $a < 1$, the graph will flatten out as it moves to the right (the x-axis will again be a horizontal asymptote), and the graph will sharply rise as it moves to the left. These types of graphs will either always increase or always decrease.

7. Logarithmic graphs: $y = \log_a x$ (a is a positive number)

The basic form of this graph will always pass through the point (1,0) – (the log of 1, for any base, always equals 0). If $a > 1$, the graph will rise as it moves to the right and will almost flatten out; as it moves to the left, it will drop sharply – the y-axis will be a vertical asymptote. If $a < 1$, the graph will drop and almost flatten out as it moves to the right; as it moves left, it will rise sharply - the y-axis will again be a vertical asymptote. These types of graphs will either always increase or always decrease.

B. Major terms associated with graphs of functions

1. Transformations

A transformation is any change made to the basic form of a function. Transformations can come in many forms – vertical shift (translation), horizontal shift, vertical reflection, horizontal reflection, period change (horizontal stretch or compression), amplitude change (vertical stretch or compression), rotations (primarily for conics, but we don't really do this in our AP Calculus course)

<u>Examples</u>: $y = (x - 2)^2 - 3$

This is a parabola moved 2 units to the right (opposite sign of the number after the x) and moved 3 units down (same sign as the number at the end)

$y = -2^x + 1$

This is an exponential growth function reflected horizontally across the x-axis and then moved 1 unit up. Usually, exponential equations pass through (0,1) and have the x-axis as their horizontal asymptote. This equation when reflected would pass through (0,-1) and still have the x-axis as an asymptote, but would be below the axis rather than above it. Then we must also translate the graph 1 unit up to get our final graph. This means that the actual graph will pass through (0,0) and have y = 1 as its asymptote, and the graph will be located below the asymptote.

$y = \cos(2x)$

This is the basic cos curve which begins at (0,1) and moves through (π/2,0) and then down to (π,-1) and back up through (3π/2,0) and finally to (2π,1) and then repeats that cycle both left and right forever. The 2 in the original equation means that the cycle is repeated twice instead of once from 0 to 2π. Therefore, the new graph will pass through (0,1) and move through (π/4,0) and then down to (π/2,-1) and back up through (3π/4,0) and finally to (π,1) and then repeat that cycle.

2. Asymptotes

An asymptote is a line that a graph gets extremely close to without ever actually touching the line. Asymptotes can be vertical, horizontal, or slant.

Vertical asymptotes most frequently occur when there is a fraction when an x-value gives a value of 0 in the denominator. Vertical asymptotes also occur with logarithmic functions.

Example: $y = \frac{x+3}{x^2-2x-8}$ Asymptotes: x = 4 and x = -2

Horizontal asymptotes most frequently occur with fractions when there is a polynomial in both the numerator and the denominator. These types of asymptotes are related to limits at infinity which you learned at the end of Pre-Calculus last year. Horizontal asymptotes also occur with exponential growth/decay functions. For now, you can use your calculator to see these asymptotes.

Example: $y = \frac{3x^2-5}{2x^2+4x}$ Horizontal asymptote: $y = \frac{3}{2}$

This graph also has vertical asymptotes at x = -2 and x = 0

Slant asymptotes also occur with fractions when there is a polynomial in both the numerator and the denominator. You can find the equation of the slant asymptote using polynomial divison (long or synthetic when possible). However, I will teach you more about these during the year. For now, you can use your calculator to see this type of asymptote.

Example: $y = \frac{x^2+2x-4}{x-1}$ Slant asymptote: y = x + 3

This graph also has a vertical asymptote at x = 1

3. Domain

This refers to all of the possible x-values that can be used for a function. Domain restrictions occur when dealing with fractions, roots, logarithms, and trigonometric functions as well as other less common functions.

When dealing with fractions, any x-value that leads to a denominator of zero cannot be in the domain.

Example: $y = \frac{x^2-3x-4}{x^2+3x+2}$ Domain: $x \neq -2$ and $x \neq -1$

x = -2 will be a vertical asymptote
x = -1 will be a hole in the graph

When dealing with even roots, any x-value that leads to a negative number under the roots cannot be in the domain.

Example: $y = \sqrt{x^2 - 16}$ Domain: $x \le -4$ or $x \ge 4$

Any x value such that $-4 < x < 4$ will result in an imaginary number which does not appear on the graph of a function

When dealing with trigonometry, any x-value that leads to a zero in the denominator for the tan, cot, sec, and csc functions cannot be in the domain.

Example: $y = \tan x$ Domain: $x \neq \ldots, -\frac{\pi}{2}, \frac{\pi}{2}, \frac{3\pi}{2}, \ldots$

Putting any of these odd multiples of $\pi/2$ will result in the the denominator of the tan function (which is cos) equaling zero

When dealing with logarithms, any x-value that leads to a non-positive number in the argument of the logarithm cannot be in the domain.

$y = \log(x + 3)$ Domain: $x > -3$

Putting -3 or any smaller number in for x will result in the argument (the "x + 3" part) of the logarithmic function being non-positive which cannot happen

4. Range

Range is the possible y-values that can come out of a function.

Determining the range of a function involves knowing the basic shape of the function and then thinking through the problem.

Examples: $y = -2x^2 - 4x + 5$ Range: $y \le 7$

x-value of vertex $= -\frac{b}{2a} = -\frac{-4}{2(-2)} = -1$, so y-value is

$-2(-1)^2 - 4(-1) + 5 = 7$ and since this parabola opens down, all other y-values must be below 7

$y = 2^{x-1} - 4$ Range: $y > -4$

this is an exponential growth graph moved 1 unit to the right and 4 units down; the basic exponential growth graph has a horizontal asymptote of $y = 0$ (the x-axis), so this graph has a horiz. asymptote of $y = -4$ and all y-values must be above -4

C. Composition of functions

A composition is when one function is placed into another function. It is usually written as f(g(x)) but is sometimes written as $(f \circ g)(x)$. This means that the function g(x) is placed into the f(x) function. We must be careful with the domain and range of composite functions. We will discuss this during the year.

Examples: $f(x) = x^2 + 2x - 4$ and $g(x) = x - 5$

$$f(g(x)) = (x - 5)^2 + 2(x - 5) - 4$$
$$= x^2 - 10x + 25 + 2x - 10 - 4$$
$$= x^2 - 8x + 11$$

$f(x) = -2x^2 + 5$ and $g(x) = \sqrt{x + 1}$

$$f(g(x)) = -2(\sqrt{x + 1})^2 + 5$$
$$= -2(x + 1) + 5 = -2x - 2 + 5 = -2x + 3$$

D. Inverse functions

Inverse functions are functions that are reflections of each other in the line y = x. All functions have an inverse, though the inverse itself may not be a function (remember, a function must pass the vertical line test). When the original function passes both the horizontal and vertical line test, then the inverse function will also be a function. There is a relationship between the domain of the original function and the range of the inverse as well as between the range of the original function and the domain of the inverse.

To find an inverse function, switch x and y in the original function and then solve for y.

Find the inverse function of $y = x^3 - 8$

Switch x and y: $x = y^3 - 8$

Solve for y: $x + 8 = y^3 \rightarrow y = \sqrt[3]{x + 8}$

Find the inverse function of $y = \frac{5}{x-2}$

Switch x and y: $x = \frac{5}{y-2}$

Solve for y: $x(y-2) = 5 \rightarrow y - 2 = \frac{5}{x} \rightarrow y = \frac{5}{x} + 2$ or $y = \frac{5+2x}{x}$

E. Symmetry – odd and even functions

To find out if a function is an odd (odd functions have 180 rotational symmetry around the origin) or an even function (even functions have vertical line symmetry across the y-axis), you find f(-x). If (f-x) is the exact same as f(x), then it is an even function. If f(-x) equals –f(x), in other words it is the exact same function but has the opposite sign for every term, then it is an odd function. Functions do not have to be either odd or even, they can be neither one.

$f(x) = x^2 + 4$ $f(-x) = (-x)^2 + 4$ which equals $f(x)$ so this is even

$f(x) = x^3 + 4x$ $f(-x) = (-x)^3 + 4(-x)$ which equals $-x^3 - 4x$ so this is odd

$f(x) = x^4 + 8x$ $f(-x) = (-x)^4 + 8(-x)$ which equals $x^4 - 8x$ so this is neither

F. Logarithms

Primarily, you need to remember your logarithmic properties.
The log of 1 always equals zero.

$\log_2 1 = 0$ $\ln 1 = 0$

The log of a power can be written as a coefficient.

$\log_4 (x^5) = 5 \log_4 x$ $3 \log 4 = \log (4^3) = \log 64$

The log of a product can be written as a sum of 2 logs.

$\log (3x) = \log 3 + \log x$ $\log m + \log 4 = \log (4m)$

The log of a quotient can be written as a difference of 2 logs.

$\log \frac{x}{y} = \log x - \log y$ $\log 6 - \log 2 = \log \frac{6}{2} = \log 3$

X. Limits

This is the material that you will learn from the online video. Whenever possible, find limits by substituting the number in for x and solving. Often, you will get a denominator of 0. When this happens, use algebra to simplify the problem, then try substituting for x again.

Examples: $\lim_{x\to -2}\frac{x^2-4}{x-1}=\frac{(-2)^2-4}{-2-1}=\frac{0}{-3}=0$ $\qquad$ $\lim_{x\to 2}\frac{x^2-4}{x-2}=\lim_{x\to 2}\frac{(x+2)(x-2)}{(x-2)}=\lim_{x\to 2}(x+2)=4$

$$\lim_{x\to 1}\frac{x^4-1}{x-1}=\lim_{x\to 1}\frac{(x^2+1)(x^2-1)}{(x-1)}=\lim_{x\to 1}\frac{(x^2+1)(x+1)(x-1)}{(x-1)}=\lim_{x\to 1}(x^2+1)(x+1)=(1+1)(1+1)=4$$

$$\lim_{x\to 5}\frac{x-5}{\sqrt{x^2-25}}=\lim_{x\to 5}\frac{x-5}{\sqrt{x^2-25}}\cdot\frac{\sqrt{x^2-25}}{\sqrt{x^2-25}}=\lim_{x\to 5}\frac{(x-5)\sqrt{x^2-25}}{x^2-25}=\lim_{x\to 5}\frac{(x-5)\sqrt{x^2-25}}{(x-5)(x+5)}=\lim_{x\to 5}\frac{\sqrt{x^2-25}}{(x+5)}=\frac{0}{10}=0$$

$$\lim_{x\to 4}\frac{2-\sqrt{x}}{x-4}=\lim_{x\to 4}\frac{2-\sqrt{x}}{x-4}\cdot\frac{2+\sqrt{x}}{2+\sqrt{x}}=\lim_{x\to 4}\frac{4-x}{(x-4)(2+\sqrt{x})}=\lim_{x\to 4}\frac{-(x-4)}{(x-4)(2+\sqrt{x})}=\lim_{x\to 4}\frac{-1}{2+\sqrt{x}}=\frac{-1}{2+\sqrt{4}}=-\frac{1}{4}$$

When doing limits as x approaches infinity, divide each term by the highest power of the denominator, then simplify. When infinity is in the denominator, the fraction becomes so small that it basically equals 0.

$$\lim_{x\to\infty}\frac{2x^2-3x+4}{3x^2+5}=\lim_{x\to\infty}\frac{\frac{2x^2}{x^2}-\frac{3x}{x^2}+\frac{4}{x^2}}{\frac{3x^2}{x^2}+\frac{5}{x^2}}=\lim_{x\to\infty}\frac{2-\frac{3}{x}+\frac{4}{x^2}}{3+\frac{5}{x^2}}=\frac{2-0+0}{3+0}=\frac{2}{3}$$

$$\lim_{x\to-\infty}\frac{-6x+14}{-2x^2+4x}=\lim_{x\to-\infty}\frac{\frac{-6x}{x^2}+\frac{14}{x^2}}{\frac{-2x^2}{x^2}+\frac{4x}{x^2}}=\lim_{x\to-\infty}\frac{-\frac{6}{x}+\frac{14}{x^2}}{-2+\frac{4}{x}}=\frac{0+0}{-2+0}=0$$

$$\lim_{x\to-\infty}\frac{-6x^2-2x+7}{-3x+15}=\lim_{x\to-\infty}\frac{\frac{-6x^2}{x}-\frac{2x}{x}+\frac{7}{x}}{\frac{-3x}{x}+\frac{15}{x}}=\lim_{x\to-\infty}\frac{-6x-2+\frac{7}{x}}{-3+\frac{15}{x}}=\frac{-6(\infty)-2+0}{-3+0}=\frac{\infty}{-3}=-\infty$$

Lastly, for difficult questions that cannot be easily simplified using algebra, you can use the "Table" feature on your calculator to find y-values directly around the x-value that is in the question.

$\lim_{x\to 0}\frac{1-\cos x}{x}$ $\qquad$ Using the table feature on your calculator, you can find the values listed below. The limit appears to be 0.

x-value	-0.03	-0.02	-0.01	0	0.01	0.02	0.03
y-value	-0.015	-0.01	-0.005	error	0.005	0.01	0.015

PRACTICE PROBLEMS

I. Factoring

Factor the following polynomials

1. $12x^2 + 6x^3 - 3x$
2. $4x^{1/2} + 2x^2 + 6x^{-1/2}$
3. $5x^{-3} + 10x^2 - 25x$
4. $x^2 + 5x + 6$
5. $x^2 - 8x - 20$
6. $6x^2 - x - 12$
7. $9x^2 + 37x + 4$
8. $x^2 - 100$
9. $16x^2 - 25y^2$
10. $x^4 - 16$
11. $x^3 - 64$
12. $8a^3 - b^9$
13. $54a^5 + 2a^2$
14. $x^3 + 2x^2 + x + 2$
15. $4x^3 + 2x^2 - 6x - 3$
16. $a^3 + 7a^2 - 4a - 28$

II. Finding roots/zeroes without a calculator

1. $4x - 28 = 0$
2. $2x^2 - 32 = 0$
3. $3x^4 - 75 = 0$
4. $x^2 + 5x = 14$
5. $2x^2 + x - 10 = 0$
6. $x^3 + 5x^2 = 9x + 45$
7. $5x^2 + 6x - 2 = 0$
8. $-2x^2 + 6x = 2$
9. $3x^3 + 24 = 0$
10. $x^3 - 20 = 7$

III. Finding roots/zeroes with a calculator.

1. $\ln (x^2 - 2x) = 0$
2. $x^3 - 2x + 1 = 0$
3. $\sqrt{x^2 - 4} = 1$
4. $\log (x + 3) + x^3 - 2x = 0$
5. $x^4 + 7x^3 + x^2 - 9x - 4 = 0$

IV. Finding intersection points without a calculator

1. $y = 2x + 5$
   $y = 4x - 1$

2. $2x + 3y = 8$
   $5x - 4y = -26$

3. $y = x - 1$
   $x^2 + y^2 = 4$

4. $y = x^2 + 2x - 10$
   $y = -x^2 - 6x + 14$

V. Finding intersection points with a calculator

1. $y = x^2 - 3x - 4$
   $y = \sin x$

2. $y = \sqrt{x + 6} - 2$
   $y = |2x - 1|$

3. $y = \log(3x^2 + 2) - 5$
   $y = -\sqrt{49 - x^2}$

VI. Formulas - find the following for each set of points

1. Slope: (-2,5) & (3,2)

2. Slope: (3,-7) & (6,-2)

3. Distance: (-5,5) & (3,1)

4. Distance: (5,4) & (4,-3)

VII. Trigonometry

1. Simplify: $\frac{1-\sin^2 x}{\sin^2 x}$

2. Simplify: $\frac{\sec^2 x + \sec^2 x \cot^2 x}{1-\sin^2 x}$

3. $\sin\frac{2\pi}{3} + \cos\frac{\pi}{4}$

4. $\frac{\tan\frac{7\pi}{6}}{\sin\frac{2\pi}{3}}$

5. $\csc\frac{3\pi}{4} - \cot\frac{5\pi}{6}$

6. Identify the amplitude & period of this function.

   $y = 2 - 4\cos(3\theta - \frac{\pi}{4})$

VIII. Line Equations

1. Find the equation of the line that passes through (-4,5) and (3,2).

2. Find the equation of the line that is parallel to $4x + 3y = 12$ and passes through (-1,-1).

3. Find the equation of the line that is goes through (3,5) and is perpendicular to $2x - y = 9$.

IX. Functions

1. Explain the transformation of the following function.

   $y = |x - 3| + 4$

2. Write the equation of a square root function that is reflected across the x-axis and is then translated 4 units up.

3. Find the vertical asymptotes of the following function.

   $y = \dfrac{x^2 - 1}{x^2 + 3x - 4}$

4. Find the vertical asymptotes of the following function.

   $y = \dfrac{x^2 + 5x + 6}{x^2 - 6x - 16}$

5. State the domain of the following function.

   $y = \sqrt{9 - x^2}$

6. State the range of the following function.

   $y = 4^{x-2} + 5$

7. Find the inverse of the following function.

   $y = 3x - 8$

8. Find the inverse of the following function.

   $y = \log(x - 4)$

9. Find the inverse of the following function.

   $y = \cos x - 5$

10. Find the composition.

    $f(x) = x^2 + 2x - 9$
    $g(x) = x - 1$

    $f(g(x)) =$

11. Find the composition.

    $f(x) = \log(x^2 + 2)$
    $g(x) = \sqrt{3 - x}$

    $f(g(x)) =$

12. Find the composition.

    $f(x) = \ln x$
    $g(x) = e^{2x}$

    $f(g(x)) =$

13. Simplify the following.

    $\log_2 (4x^2)$

14. Simplify the following.

    $\log_4 \dfrac{2}{\sqrt{x}}$

15. Simplify the following.

    $\ln \dfrac{2a^3}{b^4}$

X. Limits

1. $\lim_{x\to 3} \frac{2x - 3}{x - 4}$

2. $\lim_{x\to 4} \sqrt[3]{x + 4}$

3. $\lim_{x\to 0} \frac{x^2 - 3x}{x}$

4. $\lim_{x\to 1} \frac{x^3 - x}{x - 1}$

5. $\lim_{x\to 2} \frac{2 - x}{x^2 - 4}$

6. $\lim_{x\to 5} \frac{\sqrt{x + 4} - 3}{x - 5}$

7. $\lim_{x\to -2} \frac{x^2 - 2x - 8}{x^2 - 4}$

8. $\lim_{x\to 0} \frac{\sqrt{3 + x} - \sqrt{3}}{x}$

9. $\lim_{x\to \infty} \frac{2x^2 - 3}{x^3 - 1}$

10. $\lim_{x\to \infty} \frac{x^2 - 3x + 7}{3x^2}$

11. $\lim_{x\to \infty} \frac{2x^2 - 3}{8x + 1}$

12. $\lim_{x\to -\infty} \frac{5x^2 - 3x}{-4x^2 - 2x + 5}$

13. $\lim_{x\to -\infty} \frac{2x^3 - 3x^2 + 5}{x^2 + 3x + 2}$

14. $\lim_{x\to \infty} \frac{5x^{1/2} - 3}{\sqrt{x} + 4}$

Use a table to evaluate the following limits.

15. $\lim_{x\to 2} \frac{x^5 - 32}{x - 2}$

16. $\lim_{x\to 0} \frac{\sin^2 x}{x}$

SOLUTIONS TO PRACTICE PROBLEMS

I. Factoring

Factor the following polynomials

1. $3x(4x + 2x^2 - 1)$

2. $2x^{-1/2}(2x + x^{5/2} + 3)$

3. $5x^{-3}(1 + 2x^5 - 5x^4)$

4. $(x + 3)(x + 2)$

5. $(x - 10)(x + 2)$

6. $(2x - 3)(3x + 4)$

7. $(9x + 1)(x + 4)$

8. $(x - 10)(x + 10)$

9. $(4x - 5y)(4x + 5y)$

10. $(x - 2)(x + 2)(x^2 + 4)$

11. $(x - 4)(x^2 + 4x + 16)$

12. $(2a - b^3)(4a^2 + 2ab^3 + b^6)$

13. $2a^2(3a + 1)(9a^2 - 3a + 1)$

14. $(x^2 + 1)(x + 2)$

15. $(2x^2 - 3)(2x +1)$

16. $(a - 2)(a + 2)(a + 7)$

II. Finding roots/zeroes without a calculator

1. $x = 7$

2. $x = 4$ or -4

3. $x = \pm\sqrt{5}$

4. $x = 2$ or -7

5. $x = 2$ or $-\frac{5}{2}$

6. $x = 3$ or -3 or -5

7. $x = \frac{-3 \pm \sqrt{19}}{5}$

8. $x = \frac{3 \pm \sqrt{5}}{2}$

9. $x = -2$

10. $x = 3$

III. Finding roots/zeroes with a calculator.

1. $x = -0.414$ or 2.414

2. $x = -1.618$ or 0.618 or 1

3. $x = -2.236$ or 2.236

4. $x = -1.459$ or 0.267 or 1.220

5. $x = -6.661, -1, -0.512, 1.173$

IV. Finding intersection points without a calculator

1. (3,11)

2. (-2,4)

3. $\left(\frac{1+\sqrt{7}}{2},\frac{-1+\sqrt{7}}{2}\right)$ and

$\left(\frac{1-\sqrt{7}}{2},\frac{-1-\sqrt{7}}{2}\right)$

4. (-6,14) and (2,-2)

V. Finding intersection points with a calculator

1. (-0.846,-0.748) and (3.864,-0.661)

2. (0.25,0.5) and (0.804,0.608)

3. (-6.367,-2.908) and (6.367, -2.908)

VI. Formulas – find the following for each set of points

1. $-\frac{3}{5}$

2. $\frac{5}{3}$

3. $4\sqrt{5}$

4. $5\sqrt{2}$

VII. Trigonometry

1. $\cot^2 x$

2. $\sec^4 x \csc^2 x$ or

$\frac{\csc^2 x}{\cos^4 x}$

3. $\frac{\sqrt{3} + \sqrt{2}}{2}$

4. $\frac{2}{3}$

5. $\sqrt{2} + \sqrt{3}$

6. amplitude = 4

period = $\frac{2\pi}{3}$

VIII. Line Equations

1. $y = -\frac{3}{7}x + \frac{23}{7}$

2. $y = -\frac{4}{3}x - \frac{7}{3}$

3. $y = -\frac{1}{2}x + \frac{13}{2}$

IX. Functions

1. It is the absolute value graph moved 3 units to the right and 4 units up.

2. $y = -\sqrt{x} + 4$

3. $x = -4$ is asymptote

   $x = 1$ is not an asymptote, but it is a hole in the graph

4. $x = 8$ is an asymptote

   ($x = -2$ is a hole)

5. $-3 \le x \le 3$

6. $y > 5$

7. $y = \frac{x+8}{3}$ or $\frac{1}{3}x + \frac{8}{3}$

8. $y = 10^x + 4$

9. $y = \cos^{-1}(x + 5)$ or

   $y = \arccos(x + 5)$

10. $f(g(x)) = x^2 - 10$

11. $f(g(x)) = \log(5 - x)$

    This one has an interesting domain, we will discuss it during the year.

12. $f(g(x)) = 2x$

13. $2 + 2\log_2 x$

14. $\frac{1}{2} - \frac{1}{2}\log_4 x$

15. $\ln 2 + 3\ln a - 4\ln b$

X. Limits

1. -3

2. 2

3. -3

4. 2

5. $-\frac{1}{4}$

6. $\frac{1}{6}$

7. $\frac{3}{2}$

8. $\frac{1}{2\sqrt{3}}$

9. 0

10. $\frac{1}{3}$

11. ∞

12. $-\frac{5}{4}$

13. $-\infty$

14. 5

Use a table to evaluate the following limits.

15. 80

16. 0